40. Mitjançant va l'internet. Elecciona de l'occió

$$N_r(x,y) = \#(m,n) \in B_r(x,y).$$

$(m,n) \in B_r(x,y) \Leftrightarrow (x,y) \in B_r(m,n)$. Per tant $N_r(x,y) =$

$$= \sum_{(m,n)\in\mathbb{Z}^2} \mathbb{I}_{B_r(m,n)}(x,y) \qquad \left(\mathbb{I}_{B_r(m,n)}(x,y) = \begin{cases} 1 & \text{si } (x,y) \in B_r(m,n) \\ 0 & \text{altrament} \end{cases} \right).$$

$$\int_0^1\int_0^1 N_r(x,y)\,dxdy = \int_0^1\int_0^1 \sum_{(m,n)\in\mathbb{Z}^2} \mathbb{I}_{B_r(m,n)}(x,y)\,dxdy = \sum_{(m,n)\in\mathbb{Z}^2} \int_0^1\int_0^1 \mathbb{I}_{B_r(m,n)}(x,y)\,dxdy$$

$$= \sum_{(m,n)\in\mathbb{Z}^2} \int_{\mathbb{R}^2} \mathbb{I}_{[0,1]^2}\cdot \mathbb{I}_{B_r(m,n)}\,dxdy = \sum_{(m,n)\in\mathbb{Z}^2} \int_{\mathbb{R}^2} \mathbb{I}_{B_r(m,n)\cap[0,1]^2} =$$

$$= \sum_{(m,n)\in\mathbb{Z}^2} \text{Area}\{B_r(m,n)\cap([0,1]\times[0,1])\} = \sum_{(m,n)\in\mathbb{Z}^2} \text{Area}\{B_r(0,0)\cap([-m,-m+1]\times[-n,-n+1])\}$$

per cada $(m,n)\in\mathbb{Z}^2$, desplacem l'origen de $(0,0)$ a (m,n)

$$= \text{Area}\left\{ B_r(0,0) \cap \left(\bigcup_{(m,n)\in\mathbb{Z}^2} [-m,-m+1]\times[-n,-n+1] \right) \right\} = \text{Area}\{B_r(0,0)\cap\mathbb{R}^2\}$$

$$= \text{Area}\{B_r(0,0)\} = \pi r^2.$$