A Todo Gauss - PS4

Dimensió finita:

$$(A-B)C = BA^{-1} \iff AC - BC = BA^{-1} \iff AC = B(C+A^{-1}) \quad (1)$$

Tenim:

$$\underset{CA(C+A^{-1})}{\underset{\shortparallel}{CAC + C}} = \underset{(C+A^{-1})AC}{\underset{\shortparallel}{CAC + C}}$$

$$\Rightarrow BCA(C+A^{-1}) = B(C+A^{-1})AC \overset{(1)}{=} ACAC$$

$$\Rightarrow BCAC + BC = ACAC \quad (2)$$

Considerem $(BC\cancel{A} - AC + Id)(AC + Id) =$

$$= BCAC + BC - ACAC - \cancel{AC} + \cancel{AC} + Id \overset{(2)}{=} Id$$

Com A, B, C tenen dimensió finita:

$$(BC - AC + Id)(AC + Id) = Id \iff (AC + Id)(BC - AC + Id) = Id$$

$$\Rightarrow ACBC - ACAC + \cancel{AC} + BC - \cancel{AC} + \cancel{Id} = \cancel{Id}$$

$$\Rightarrow ACBC - ACAC + BC = 0 \quad \cancel{\iff ACBC + BC}$$

$$\iff (ACB - ACA + B)C = 0$$

• Si $C \not\equiv 0 \Rightarrow ACB - ACA + B = 0$

$$\iff CB - CA + A^{-1}B = 0$$

$$\iff A^{-1}B = C(A-B) \checkmark$$

• Si $C \equiv 0 \Rightarrow (A-B)C = 0 = BA^{-1} \Rightarrow B \equiv 0$

$$\Rightarrow C(A-B) = 0 = A^{-1}B \checkmark$$

Dimensió infinita:

En aquest cas no és cert:

Considerem A, B, C endomorfismes a $\mathcal{C}^\infty([0,1]) = E$

$$A: E \to E,\quad f \mapsto f \qquad , \qquad B: E \to E,\quad f \longmapsto f - \frac{df}{dx} \qquad , \qquad C: E \to E,\quad f \longmapsto \int_0^x f - f$$

Són clarament endomorfismes. (i lineals)

$B \circ A^{-1}$

$A^{-1}B = B$ ✓

① $(A - B)(f) = f - \left(f - \frac{df}{dx}\right) = \frac{df}{dx}$

$$\Rightarrow (A-B) \circ C(f) = (A-B)\left(\int_0^x f - f\right) = \frac{d}{dx}\int_0^x f - \frac{df}{dx} = f - \frac{df}{dx} = B(f)$$

② $$C \circ (A - B) = C\left(\frac{df}{dx}\right) = \int_0^x \frac{df}{dx} - \frac{df}{dx} \overset{?}{=} f - \frac{df}{dx} = B(f)$$

$$\Leftrightarrow \int_0^x \frac{df}{dx} = f$$

Però en general:

$\int_0^x \frac{df}{dx} \neq f$ per exemple la funció $f \equiv 1$ ($f \in \mathcal{C}^\infty[0,1]$ ✓)

$\Rightarrow \int_0^x 0 = 1$!!! No és cert: $(A-B)C = BA^{-1} \Rightarrow C(A-B) = A^{-1}B$.