

sigui  $s_n^m$  definit de la següent forma:

$$s_n^n = 1 \quad \forall n \geq 0$$

$$s_{n+1}^n = a_n \quad \forall n \geq 0$$

$$s_n^m = a_{n-1} s_{n-1}^m + s_{n-2}^m \quad \forall n \geq m+2 \quad s_n^m \text{ indefinit si } n \leq m$$

notis que  $s^0 = \alpha$  i  $s^1 = \beta$

a més,  $s_n^m = s_n^{m+1} a_m + s_n^{m+2}$   $\forall n \geq m+2$ , es veu inductivament:

$$\text{cas } n=m+2 \rightarrow s_{m+2}^m = a_{m+1} s_{m+1}^m + s_m^m = a_{m+1} a_m + 1 = s_{m+2}^{m+1} a_m + s_{m+2}^{m+2} \checkmark$$

$$\text{cas } n=m+3 \rightarrow s_{m+3}^m = a_{m+2} s_{m+2}^m + s_{m+1}^m = a_{m+2} (a_{m+1} a_m + 1) + a_m = (a_{m+2} a_{m+1} + 1) a_m + a_{m+2}$$

$$= s_{m+3}^{m+1} a_m + s_{m+3}^{m+2} \checkmark$$

(totes les igualtats usen la definició de  $s$ )

cas  $n \geq m+3$

I.H.

$$\begin{aligned} s_n^m &= a_{n-1} s_{n-1}^m + s_{n-2}^m \stackrel{\text{I.H.}}{=} a_{n-1} (s_{n-1}^{m+1} a_m + s_{n-1}^{m+2}) + s_{n-2}^{m+1} a_m + s_{n-2}^{m+2} = \\ &= (a_{n-1} s_{n-1}^{m+1} + s_{n-2}^{m+1}) a_m + a_{n-1} s_{n-1}^{m+2} + s_{n-2}^{m+2} = s_n^{m+1} a_m + s_n^{m+2} \checkmark \end{aligned}$$

$$\text{si } p | s_n^m \text{ i } p | s_n^{m+1} \Rightarrow p | \underbrace{s_n^{m+1} a_m + s_n^{m+2}}_{p | s_n^m} - \underbrace{a_m s_n^{m+1}}_{p | s_n^{m+1}} = s_n^{m+2}$$

iterant aquesta implicació, arribem a  $p | s_n^{m+(n-m)}$  aka  $p | s_n^n = 1$

$$\Downarrow \\ p=1$$

en concret, posant  $m=0$ , tenim  $p | \alpha_n$  i  $p | \beta_n \Rightarrow p=1$

aka  $\alpha_n$  i  $\beta_n$  son coprimers