$\boxed{\text{P1887}}$ Basta ver que:

$$\sum_{k=0}^{2n} \frac{1}{k!} = \sum_{k=0}^{\infty} \frac{1}{k!} - \frac{1}{(2n+1)!} - \frac{1}{(2n+2)!} \sum_{k=0}^{\infty} \frac{1}{\prod_{j=2n+3}^{2n+2+k} (j)} \overset{1 \leq \sum_{k=0}^{\infty} \frac{1}{\prod_{j=2n+3}^{2n+2+k}(j)} < \sum_{k=0}^{\infty} \frac{1}{2^k} = 2}{=}$$

$$= \sum_{k=0}^{\infty} \frac{1}{k!} - \frac{1}{(2n+1)!} - \frac{1}{(2n+2)!} O(1) = e - \frac{1}{(2n+1)!} \left(1 + O\left(\frac{1}{2n+2}\right)\right) =$$

$$= e - \frac{1}{(2n+1)!} \left(1 + O\left(\frac{1}{n}\right)\right)$$

$$\sum_{k=0}^{2n} \frac{(-1)^k}{k!} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} + \frac{1}{(2n+1)!} - \frac{1}{(2n+2)!} \sum_{k=0}^{\infty} \frac{(-1)^k}{\prod_{j=2n+3}^{2n+2+k} (j)} \overset{\text{valor absoluto del sumatorio final } \leq 2}{=} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} + \frac{1}{(2n+1)!} - \frac{1}{(2n+2)!} O(1) =$$

$$= e^{-1} + \frac{1}{(2n+1)!} \left(1 - O\left(\frac{1}{2n+2}\right)\right) = e^{-1} + \frac{1}{(2n+1)!} \left(1 + O(n)\right)$$

Por tanto:

$$\lim_{n \to \infty} \left((2n+1)! \left(\sum_{k=0}^{2n} \frac{1}{k!} \cdot \sum_{k=0}^{2n} \frac{(-1)^k}{k!} - 1 \right) \right) =$$

$$= \lim_{n \to \infty} \left((2n+1)! \left[\left(e - \frac{1}{(2n+1)!} \left(1 + O\left(\frac{1}{n}\right)\right)\right) \left(e^{-1} + \frac{1}{(2n+1)!} \left(1 + O\left(\frac{1}{n}\right)\right)\right) - 1 \right] \right) =$$

$$= \lim_{n \to \infty} \left((2n+1)! \left[1 + \frac{1}{(2n+1)!} \left(e - e^{-1} - \frac{1}{(2n+1)!} + O\left(\frac{1}{n}\right)\right) - 1 \right] \right) =$$

$$= \lim_{n \to \infty} \left((2n+1)! \frac{1}{(2n+1)!} \left(e - e^{-1} + O\left(\frac{1}{n}\right)\right) \right) =$$

$$= \lim_{n \to \infty} \left(e - e^{-1} + O\left(\frac{1}{n}\right) \right) = \underline{\underline{e - e^{-1}}} = \frac{e^2 - 1}{e}$$